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Metaheuristics For An Optimization Problem (In The Game Of Bridge) As Strategies

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Abstract

In this paper a formalization of bridge declarer's play is proposed for the purposes of theoretical and simulative investigations of heuristics. Several metaheuristics based on different aspects of the formal model are proposed.

Keywords: Expectation, Random event, Scale, Game scoring, Monte Carlo simulation

1 Introduction

In the past, games like chequers, chess and others have served as inspirations for various models of artificial intelligence.

The game of bridge can also be treated as an intellectual exercise. But there is a huge emotional tightness in this game either. Both aspects can be and often are closely related. Therefore, an investigation from the AI point of view may be considered as another interesting challenge and not only for elaborating computational methods.

Bridge is a classical and prestigious game with the main features described in: `http://en.wikipedia.org/wiki/Contract_bridge`

In the conceptual map of bridge declarer's play has a very special place. Any theory of and for bidding should be finally checked by a sort of simulation based on declarer's play.

Good players during the bidding often try to imagine the forthcoming declarer's play.

It also should be taken into account while a defender plans the first lead. Theory of declarer's play is obviously also very relevant for planning the last element of bridge: the defense.

Game of Sudoku (which actually is rather an intellectual exercise than game) can be compared with declarer's play in bridge. Lately in the computer science literature there appeared several mathematically based algorithms for solving Sudoku problems, see e.g.: Yue and Lee 2006. It is not very fast development taking into account that game has existed (and is popular in many countries) about 20 years.

Bridge at expert level has been developed from the beginning of XX century. "Mathematization" of it is still behind the Sudoku, nevertheless... It seems that existing mathematical descriptionsof declarer's play were motivated by specific needs without shaping the general panorama. ...

2 Optimality of declarer's play

Assume a contract for a given deal (hand) was established and a first lead was done. Declarer wants to fight successfully against the opponents, who generally are rational and have certain level of expertise. This level can (and often must) be assessed by declarer. Such an estimation has normally certain error or "standard deviation".

Declarer to pretend for any kind of optimality must, of course, take into account the system of scoring for a given hand that is played. It worth adding that any kind of existing scoring is based on the classical: Vanderbilt rubber scoring that has more than 80 years. For more details and proposals see Ramer 2007. Computation of the actual score only begins with Vanderbilt's value that is assigned to result of the deal. We will continue this theme further...

Declarer has to choose several optimal cards during the playing process. k will index the positions declarer has to take decision in. It follows from the play rules that formally k runs from 1 to 24. Normally at the high level of bridge expertise it runs shorter, however, because either declarer or one of the defenders claims how the resting (last) tricks will be played and distributed. Such claim normally stops the playing process of a given handAlso positions with only one card that can be played in a given trick are not subject to optimality considerations, obviously.

Assume $C(k)$ is the set of all possible cards, which can be played in actual position. It applies either for the dummy (that is tabled) or for declarer's closed hand.

After an analysis (in position k) declarer can establish all possible numbers of tricks that can be taken by him at the end of deal. He tries and will try in next positions to maximize the final number of tricks (among those possible), while opponents (defenders) will make everything to complicate his tasks in the deal.

Let $I(k)$ stands for (all) those possible final numbers of tricks in the position k (≤ 24). $I(k)$ is a subset of $\{0, 1, \ldots, 13\}$.

The task of declarer in the k-th position is to maximize for c the following expected value:

$$E[\sum_{i \in I(k)} P_i(c) Z_i(c)] \tag{1}$$

where c is the card to be played (c is an element of $C(k)$), $Z_i(c)$ is a score that will be finally obtained by declarer's side if this side will win i tricks. Probability of such result is equal to $P_i(c)$.

This criterion should be maximized in a discrete set $C(k)$. It means that we as declarer should find a card c that is best in the above sense. Very often there exist several cards in the set: $C(k)$ that are equally optimal. Some of them can be even equivalent. E.g. if there is Queen and Jack of hearts in dummy it always doesn't matter which of them is played. The same configuration in closed hand is not equivalent, because if Queen is played, both defenders can (or must) consider possibility that Jack is in hands of partner.

Scale of scores Z depends clearly on the scoring system of the actual hand.

In most cases all components that appear in (1) are random variables. Distributions of those variables are unknown for declarer. However he always has some knowledge about those distributions. For simplicity we will assume below that $I(k)$ is not a random set.

There are 2 "natural" restrictions on the Z and P sequences of random variables for a given k. These restrictions always remain valid in random circumstances that characterize situation of declarer.

2.1 First set of restrictions

Θ is the set of elementary eventsthat is common to all random variables: $Z_i(c)$ for given k and i. Θ is therefore a third domain of the function $Z_i(c)$. First domain (for arguments) refers to index and is $I(k)$ while the second (for cards) is $C(k)$. It means that: function with 3 arguments: $Z_i(c)\{e\}$ with $e \in \Theta$ is a fixed numeric value within the scale range of scores Z.

For given c and e:

$$Z_i(c)\{e\} \leq Z_{i+1}(c)\{e\}$$

with appropriate values of i.

Sense of this set of restrictions is the following.

It should be considered that other declarer players, who play the same hand on other tables, obtain certain distribution of scores. This distribution should be imagined (but very rarely strictly assessed) by declarer at actual: k-th stage of play.

For given: card c to be played, final number: i of tricks to be taken by declarer, e represents incertitude of declarer about the value i tricks can obtain as a score for his pair side. This score depends on the system of scoring and the results obtained on other tables.

Now with all these things fixed the above inequality makes sense and is even obvious.

2.2 Second set of restrictions

Somewhat similar considerations can be made for random variables that represent probabilities: $P_i(c)$. This time i is not fixed.

Assume k and c are given. Π is a set of elementary random events that influence random variables: $P_i(c)$. Declarer does not know exactly the probabilities that correspond to the final number of tricks. Elementary event is a declarer's concept about how these probabilities can distribute after the specific card c is played by declarer.

If $f \in \Pi$ then

$$\sum_{i \in I(k)} P_i(c)\{f\} = 1$$

The sequence $P_i(c)$ forms therefore the Dirichlet process (see e.g.: Rasmussen 2000).

2.3 General practical consequences of the optimization formula

One can imagine (non-trivial) situations when a good player or a well-programmed automatic player can estimate all distributions of random variables that form the above criterion (1). Still, unless estimations are actually (extremely) very good, a card c that maximizes criterion with estimated (and not real) distributions can not be treated as the optimal one. It can be assumed, therefore, that in most non-trivial situations declarer should use a sort of heuristic and/or metaheuristic to choose appropriate cards. Moreover, a rational condition states that at least 2 appropriate heuristics should be taken into account for given non-trivial position.

If both heuristics lead to the same card c, then problem can be treated as (practically) solved. It is because optimization is realized in a small discrete set: $C(k)$. More considerations on this practical solution will be presented below.

If optimal sets (of cards) for 2 heuristics are disjoint it is where the interesting expert analyzes begin. Very often game starts to be very emotional for declarer or opponents. To some extent the situation with 2 "best" solutions is paradoxical.

There exists a bitter aphorism within the bridge community:

"It is not true that the deal is consisted of 4 components: bidding, first lead, declarer's play and defense. There is a fifth component, a most important one: the evaluation of partner, after the last trick!"

3 Properties of criterion formula

One may think that numeric scale of scores Z_i plays at least an important role in the process of choosing optimal cards in all positions. Possible values of Z_i are those that actually indicate the numeric range of criterion in (1). If values Z_i are, say, in interval [10, 90] then all possible values of criterion must belong to the same interval.

Let's consider the following proposition.

To formulate the result we must fix k as a number of declarer's position. The property will be nevertheless valid for all k's.

$Z_i(c)$ as random variable is a (measurable) function defined on a set of elementary random events Θ.

For all values $Z_i(c)$ is replaced by $Y_i(c)$

$$Y_i(c) = aZ_i(c) + b$$

i.e. this equation holds for all elementary events in Θ, $i \in I(k)$, and for all c in $C(k)$.

In other words we introduce a new scale for scores. Optimality problem (1) can be rewritten for new scoring.

The result says that for any numeric a and b ($a > 0$) an optimal card c for original scoring will also be optimal for the scoring linearly rescaled with parameters a and b.

4 Why such metaheuristics?

These are some short general remarks for introduction of the forthcoming methodology.

As was stated earlier bridge is a game with a long history and a very large literature. Declarer's play, and defense that is strictly connected with it, has hundreds methods, maneuvers, and tips described (and even not described ...) in the literature. There are hundreds of thousands of examples presented by experts on how those methods can be applied in the real deals.

This suggest a special approach to solving the above optimization problem.

More formal and/or mathematical methods that deal with similar optimization problems seem to be too general, so they will not be described here. Among those are:

1. General Monte Carlo simulations (Greenwood and Wefelmeyer, 2003).
2. Simulated annealing (Srichander, 1995).
3. Algorithms of tactics (Giunchiglia and Traverso, 1996)
4. General metaheuristics in combinatorial optimization (Blum and Roli, 2003)

Our approach is the following. As was stated as a practical consequence of (1), in a problem situation at least the two methods should be applied. Those methods will try to confirm each other.

Formally it is enough to apply some traditional, professional bridge techniques (among those hundreds existing) and compare the results. Once we have the general optimization problem stated, some guidelines for declarer are possible. Since traditional methods already confirmed their usefulness, they can be treated as heuristics. The guideline for use of heuristics is an important aspect of all declarer's play metaheuristics listed below.

5 Metaheuristic 1

There exists a "most natural" way to obtain some more constructive procedure or actually the set of procedures. A first step in this way is the following first simplification of the formula (1).

Assuming k and $i \in I(k)$ fixed, let's replace:

$$Z_i(c) := i$$

in the optimization formula.

It simplifies (1) in 2 senses.

1. There remains only one series of random variables in the optimization formula: the probabilities. Scores become fixed.
2. The resulting formula can be interpreted as an expected number of tricks that can be taken by declarer. Contrary to the expected final score – value that appears in the original criterion (1) – the simplified sum has a constructive feature. Final number of tricks can be estimated quite well by declarer during the play.

This metaheuristic looks promising because it always (i.e. from the first to the last decision positions) reminds declarer that he must maximize the expected number of tricks taken by his side.

The more realistic evaluation of this metaheuristic follows, however, from the "linear" property of basic criterion. The following reasoning shows a main disadvantage of guidelines behind metaheuristic 1.

This metaheuristic would be a first candidate as assistant for declarer if one can assume that differences between subsequent (for i) values of $Z_i(c)$ are comparable. It is generally not so, however. In most or at least in very many cases there exists a desired number of tricks, say j tricks, that should be taken by declarer. It means that difference between: $Z_j(c)$ and $Z_{j-1}(c)$ is significantly larger than all other subsequent differences. This significance is measured in expected values since $Z_i(c)$ are random variables.

Scaling $Z_i(c) = i$ means that all subsequent differences are equal.

Special focus of metaheuristic 1 is for conserving declarer's main source or sources of tricks. Also everything must be done to disturb opponents to activate their main sources of tricks. There are tens of techniques for that. They partially depend on the type of contract, which is actually played. Most of them are connected with conserving (own) and cutting (defenders') communication within pairs and keeping trumps control. Some of those techniques are typical for trumps contracts while some other are typical for no trumps. Holdup is e.g. a very typical maneuver for the no trump contracts. It tries to cut communication between defenders. For trump contracts holdup can have importance for so called cross-ruffing.

6 Metaheuristic 2

There exists a radical solution for quite common situation when there is a desired number of tricks for declarer. Let's simplify scores in (1) this way:

$$Z_i(c) := 0 \text{ for } i < j, \text{ where } j \text{ is a desired number}$$

$$Z_i(c) := 1 \text{ for } i \geq j$$

This metaheuristic tries to maximize probability to realize the desired number of tricks. Such strategy has a long history in the theory of bridge. These are the main features of this strategy.

Declarer has to find at least 2 "lines" of playing his contract in given position k. He estimates probabilities of those lines. Then he chooses the line with the highest probability of success.

This strategy needs 3 components that must be successfully solved.

1. Declarer must know appropriate a priori probabilities of different card configurations.
2. This probabilities must be properly adjusted, taking into account entire information available for declarer in the position k.
3. Probabilities of the "lines" must be correctly computed.

7 Relation to existing methods and strategies

Radical differences in forms of simplifications of scores: $Z_i(c)$ led to significant differences in playing strategies. Also types of computations that are needed in two metaheuristics above are of different nature.

It is important to state that existing methods of scoring (bridge deals) take (to some extend) into account a difference that we have established formally.

Real methods of scoring bridge events can be divided in two general types. We will call them here: MPs and IMP. First of them is connected rather with metaheuristic 1, while the IMP scoring is oriented rather for the second metaheuristic.

Richard Pavlicek is one of the leading experts in theory and teaching of bridge. In his site there are 18 lessons on basic methods for declarer's play (see: `http://www.rpbridge.net/rppl.htm`).

Only one of those lessons focuses on declarer's play differences between IMP and MPs scoring. It means that at the level of simple heuristic it is difficult to say whether a given specific play technique is dedicated for maximizing expected number of tricks or for maximizing probability of realizing the desired number of tricks.

8 Metaheuristic 3

As it will be seen this is a conditional counterpart of the metaheuristic 1.

Besides the probabilistic events that appear during the subsequent stages of declarer's planning, several "deterministic" considerations should be taken into account, too.

8.1 Cards that are conditionally optimal

Assume we (as declarer) are in the k-th position. Put U_i as a symbol of one of the possible distributions of cards between opponents (in the actual position). It can be easily proved that there exists: a set $M(U_i)$ being subset of $C(k)$, which is optimal in the following sense as choices for declarer.

Assume declarer and both defenders know all remaining cards, i.e. they know certain U_i. Such situation is called by bridge players: double-dummy.

By playing a card from $M(U_i)$ declarer maximizes his side's number of tricks assuming all players make no error in (non-realistic) double-dummy conditions.

We look for all (other) defenders' card partitions W_j such that:

$$M(U_i) \cap M(W_j) \neq \emptyset$$

where $\emptyset$ is the empty set.

Let's call Di the collection of partitions with the above property. Nonempty set:

$$\cap_{D_i} M(Wj) \tag{2}$$

contains cards that are optimal conditionally for D_i

8.2 Division of the set of opponents partitions

We define a (finite) sequence $\{D_n\}$ in the following way.

Assume U_1 is an arbitrary partition of cards between opponents. U_2 is arbitrary partition that, however, does not belong to D_1 . We establish D_2 and U_3 accordingly.

We continue this procedure.

Assume $D_1 + D_2 + D_3 + D_4$ forms the set of all possible partitions of cards between defenders (i.e. opponents of declarer's pair). It can be checked that D_n in such a sequence are disjoint.

First steps of metaheuristic 3 do not use the notion of probability... Moreover, there exist several (deterministic) algorithms for establishing $M(U)$. Historically first (of them) implemented, and very important for the community of bridge players and bridge tournament directors was the famous Deep Finesse computer program. (`http://www.deepfinesse.com/dfentry.html`).

8.3 Further steps of metaheuristic

Next step of the present metaheuristic is non-deterministic. It is also less straightforward algorithmically. Declarer has to calculate probabilities of four D_n–s taking into account the information that is available at stage k.

Last step is completely clear. We choose D_λ with the highest probability. Optimal move is the one that belongs to intersection (2) corresponding to a D_λ chosen.

D_λ can be treated as condition for the optimal play under this metaheuristic.

9 Metaheuristic 4

This is a competitor to metaheuristic 2 being at the same time a form of methodological synthesis of all metaheuristics.

Being conditional as the previous one it uses, however, a more probabilistic methodology.

First the unconditional expectation – like the one in metaheuristic 1 – should be estimated. This value has to be compared with the desired number of tricks (as a competitor of metaheuristic 2, this metaheuristic assumes such number to be known in position k).

Basic option for this metaheuristic arises when the expectation is smaller than a desired number of tricks with adjective "smaller" being subject to the decision of declarer. Sometimes e.g. expectation 9.8 with the desired number being 10 is diagnosed as "smaller", sometimes not.

Now declarer has to consider a sequence of pairs:

- An assumption about certain configuration of cards between opponents. This assumption should increase the expected number of tricks. Configuration means a set of opponents' partitions D. First component of the pair is the expected number of tricks under condition D.
- Probability of postulated assumption.

Of course, while considering a given pair, all necessary calculations have to be made. Decision which pair should be chosen is made taking into account both: conditional expectation and probability of assumption. Clearly pair with both highest components is the one declarer should choose. For non-trivial positions (indexed by k) it is typical, however, that one pair (say, A) is the best in the first component, while other pair (B) leads in the second component. And the best solution (as pair) is very often a pair with components, which are in between A and B.

There is, sometimes, an additional option of this metaheuristic. Assume that for the "majority" of cards configurations, expected number of tricks is greater or equal to the desired number of tricks. Now we look for partitions D that decrease the expected number of tricks. Further part of algorithm is analogous.

This metaheuristic is a sort of optimization with 2 criterions.

10 Validity

Straightforward simulation of any heuristic or metaheuristic (as solvers of problem (1)) would be very ineffective, while possible under strong restrictions. For majority of problematic positions both random vectors that appear in (1) form a very large sample space that has to be model by Monte Carlo or other method of simulation. One may (actually: must) consider different types of restrictions on the sample space. Moreover there is extremely difficult to formalize restrictions, which model behavior of certain type of players.

We will show other approach underlying its advantages. This approach summarizes all considerations presented above.

For any position (k) declarer should decide whether he has or not has a desired number of tricks defined. It's worth noting that sometimes declarer can change his point of view during the hand i.e. for different k's.. The change can go in any direction. At the beginning declarer could have a desired number of tricks and later he just fights for a maximum of tricks. Or at the beginning a desired number was 10 but later it becomes 9. Or at the beginning declarer did not know if he has a desired number or not (so he had no decision), and later he decides that definitely there is no such number etc.

For both declarer's decisions: having and not having the desired number of tricks we proposed 2 metaheuristics. First and third are oriented for the latter decision while second with fourth for the former one.

It seems that – logically – a very effective procedure for finding the best move in the actual position is the following. It is crucial that - opposite to majority of optimization problems - the set of arguments is very small: often it consists of only 2 non-equivalents cards.

Assuming declarer is firm about the fact that if there exists the desired number of tricks for him in the deal, he chooses 2 heuristics that belong to respective 2 metaheuristics in the pair. If it is not clear for declarer if there is the desired number of tricks in the deal or not the optimal method for him is the following. He has to choose 2 heuristics: one that assumes a desired number of tricks and the other that maximizes expected number of tricks.

There is no fundamental difference in implementing metaheuristics between human and automatic player. The difference arises at the level of heuristics. Of course there exist procedures that are "easier" and/or "more natural" for humans and vice versa.

It is of significant importance how "different" and "independent" are the actual procedures for a given deal. Sometimes the relation between procedures can change even for different positions of the same deal. If procedures are "sufficiently" different and/or independent then it is virtually impossible that the same move (card chosen) suboptimal for both procedures would not be the optimal. Since the proposed metaheuristics are based on different rationales there is a room for "difference" and "independence" of actual algorithms.

11 Concluding remarks

In this paper, metaheuristic approach to bridge playing has been (at least partially) formalized. It is worth noting, that though the game has been viewed from the declarer's point of view, it can be easily transferred to opponents' play strategy. There are 4 metaheuristics for defenders, therefore.

The fundamental difficulty consists in taking into account psychological factors that influence on the probabilities involved in metaheuristics (and heuristics connected with them) . We could assume rational players (playing the "optimal game"), but this is not rational. A realistic simulator program should instead model "level of expertise" via e.g. implementing a set of human-learned heuristics (there are hundreds of them) and simulate the expertise via the level of remembering a subset of these rules or error level of using them.

The simulation under general assumptions may be very computation intense, or even hard to execute, therefore some shortcuts like the heuristics "intersections" may be used to prune the space of considered cases.

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