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Granular Multilevel Rough Entropy Thresholding in 2D Domain

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Abstract

The paper addresses the problem of image segmentation in 2D domain by means of maximizing rough entropy measure in granular computing setting. Proposed 2D multilevel rough entropy thresholding extends multilevel thresholding scheme into 2D image thresholding. Proposed thresholding algorithm 2D $MRET$ has been compared with standard multilevel thresholding based on *Otsu* method. Experimental results suggest that proposed solution has proved to be equally robust as standard *Otsu* method, outperforming the latter in many experimental runs.

Keywords: Image thresholding, rough entropy, rough sets, granular computing

1 Introduction

Image segmentation presents a fundamental problem in image analysis routines. Threshold or multithreshold selection based segmentation routines constitute important area of research with many practical applications. Several illustrative examples of extensively used thresholding techniques are document image analysis, map processing, scene processing, quality inspection of materials, knowledge representation, image segmentation in general, segmentation of video images.

In practical applications, most often image regions do not depict well-defined homogeneous characteristics, so it seems naturally appropriate to use techniques that additionally incorporate the ambiguity in information for performing the thresholding operation. In recent years Pawlak (2007), the theory of rough sets has gained considerable importance with numerous applications in diverse areas of research, especially in data mining, knowledge discovery, artificial intelligence and information systems analysis. Combination of thresholding methods with rough set theory has been attempted by Pal (2005). The authors minimize the roughness value in order to perform image thresholding by optimizing an entropy measure, which they refer to as the "rough entropy of image". Improved, separate thresholding scheme based on optimizing rough entropy measure is given in Malyszko and Stepaniuk (2008) referred to as 1D $MRET$ algorithm. In the present study, extension of one dimensional rough entropy multithresholding algorithm 1D $MRET$ has been proposed into two-dimensional domain as 2D $MRET$ algorithm. Invented

2D thresholding algorithm has been further compared to standard 2D thresholding based on *Otsu* method applied to 2D images. Experimental results suggest that 2D $MRET$ thresholding results in robust segmentations that are comparable and often better to standard 2D *Otsu* segmentation schemes.

In Section 2 introductory information on image segmentation and thresholding routines has been presented. Further, in Section 3 description of bi-level rough entropy thresholding scheme is given and compared with 1D $MRET$ algorithm. Next, extension of 1D $MRET$ algorithm into 2D domain has been proposed. In Section 4 experimental setup and experimental results are given. Conclusions and future research in the area of rough entropy thresholding is given as summarization of the presented research material.

2 Image Thresholding

2.1 Image segmentation

In image segmentation determining distinct regions is closely coupled with subsequent image classification with two different approaches commonly used, namely supervised and unsupervised image segmentation. In supervised approach, the number and the concrete characteristics of the classes (e.g. means, variances etc) are given in advance and these predefined parameters are used in the training stage after which segmentation (or classification) step follows. Unsupervised segmentation and supervised segmentation (with further classification) starts by creating partition of the image data into groups by means of defining similarity measure, which values for image data are then compared and on that basis image data partitioning follows. Image segmentation routines are divided into: histogram based routines, edge-based routines, region merge routines, clustering routines and some combination of the above routines. Exhaustive overview of the segmentation methods is available in Haralick (1985). Edge detection approaches to image segmentation deal with discovering image locations where sharp changes in grey level or color are detected. The main difficulty in this type of algorithms is maintenance of the continuity of detected edges. Segments have always to be enclosed by continuous edges. However, usually disconnected or isolated edges within areas with more details have to be combined by using specialized heuristics. Region growing or merging presents an approach for image segmentation where large continuous regions or segments are detected first and afterwards, small regions are subjected to merging operation by use of a homogeneity criteria.

2.1.1 Thresholding technique categories

Image thresholding is basically interpreted and executed as separation of object (or objects) of interest from the background. The reason justifying an underlying assumption of thresholding operation is based on the observation that pixels belonging to the object are substantially different from pixels belonging to the foreground or equivalently, objects are different compared to each other. Simple thresholding presents a non-linear operation converting a gray-scale or other attributed image into a binary image with pixels assignment depending whether their values are below or above the specified threshold value T. Thresholding is a commonly used technique in image segmentation because of its fast and easy application. For this reason threshold selection is an important issue. There are two general approaches to threshold selection. One approach is based on the histogram of the image while the other is based on the gray scale information located in the local small areas. The histogram of an image contains some statistical data of the grayscale or color ingredients. Reader is referred to Sezgin (2004) for exhaustive image thresholding categories description.

3 Multilevel rough entropy thresholding for two-dimensional data

3.1 Multilevel thresholding routines

Multilevel rough entropy thresholding for two-dimensional data presents innovative approach based on thresholding scheme proposed in Malyszko and Stepaniuk (2008). Pal in Pal (2005) presented solution for bi-level thresholding by means of maximizing rough entropy measure in granular setting. This solution is designed for finding only one threshold at a time. In Malyszko and Stepaniuk (2008) Pal's algorithm has been extended from bi-level one-dimensional algorithm into iterative 1D splitting strategy and new 1D multilevel thresholding scheme $MRET$ entirely based on rough entropy maximization with optimal thresholds searched by means of evolutionary algorithm has been invented. Proposed granular 1D $MRET$ thresholding algorithm contributed in the area of finding N-levels for image thresholding. Theory of rough sets has been detailed in publications Pawlak (1991), Pawlak (2007), Stepaniuk (2001).

In present study, publication subject refers into extension of multi-level rough entropy multilevel thresholding into 2D domain thresholding. Proposed extension deals with arbitrary two images referred to later as 2D images. Taking 2D images, 2D $MRET$ algorithm computes in granular computing setting rough entropy measure for given N - thresholds in each of two dimensions. In this way, $(N+1)*(N+1)$ ranges in feature space (2D histogram) are created. In order to choose proper 2D thresholds, evolutionary algorithm is employed in the process of effective exploring of the search space . For this reason, population of S solutions is maintained, each solution consisting (N, N) thresholds, next RE rough entropy measure for processed solutions is calculated, solutions undergo selection, crossover and mutation operations creating finally child population. Additionally, elite of best solutions relative to β-index value is maintained. Procedure of creating child population from parental population is iteratively repeated predefined number of times or either other termination criteria are met. As a result, population with the best solutions is obtained, from this population the best solutions are selected as a final solution for thresholding operation. In the experiments, three separate images set have been taken into account. 2D $MRET$ thresholding has been carried out and compared with standard 2D *Otsu* thresholding. Both 1D $MRET$ and 2D $MRET$ routines process groups of neighboring pixels referred to as granules. Granules have size of GX and GY pixels in image width and height dimensions, and are shifted by GSX and GSY pixels relative to their neighboring granules. In this regard, granules are described by $G(GX, GY, GSX, GSY)$ parameters.

3.2 Granular rough entropy multithresholding algorithm $MRET$

In this Subsection, originally proposed by authors of this paper in Malyszko and Stepaniuk (2008) extension of one-dimensional iterative splitting Rough Entropy Algorithm is presented in the form of Multilevel Rough Entropy Thresholding Routine - 1D $MRET$. Algorithm tries to find solutions, it means N-thresholds - that maximize roughness measure that leads to minimal roughness of segmentation and exhibits high quality homogenous segments. In the algorithm run, granules of neighboring pixels are processed by computation of lower and upper approximation of each segment, and further calculation of segment roughness and roughness entropy. In the 1D $MRET$ algorithm, initial population of $Size$ solutions is created. Each solution or chromosome consists of N randomly created threshold values sorted in ascending order. Additionally, for each chromosome, its fitness value is calculated according to Rough_Entropy_Measure given in Algorithm 2. Next, standard evolutionary populations are created with consecutive selection, cross-over and mutation operations. Procedure is iteratively repeated until selected termination condition is met, for example until given number of populations has been created. Finally, algorithm presents the pool of solutions in the form of N threshold values each. From this solutions, the best should be selected as threshold values for thresholding. The way of calculation of rough entropy measure is presented briefly in Algorithm 2. Detailed algorithm description is given by authors Malyszko and Stepaniuk in Malyszko and Stepaniuk (2008) .

Algorithm 1: Granular Multilevel Rough Entropy based Thresholding Algorithm Flow

```
Data: Input Image
Result: Optimal Threshold Value
1. Create X population with Size random N-level solutions (chromosomes)
repeat
    forall chromosomes of X do
        calculate their rough entropy measure values
        Rough_Entropy_Measure;
    end
    create mating pool Y from parental X population ;
    apply selection, cross-over and mutation operators to Y population;
    replace X population with Y population ;
until until termination criteria ;
```

Algorithm 2: Granular Rough Entropy calculation Rough_Entropy_Measure

```
Data: Input Image, granule parameters: GX, GY, GSX, GSY ;
T_0, T_1, ..., T_{n-1}, n Input Thresholds ;
Result: Rough Entropy Measure
1. Create the set Int of intervals generated according to Input Thresholds
 T(1) =< 0, T_0 >, T(2) =< T_0 + 1, T_1 > ... T(n) =< T_{n-1} + 1, 255 >
2. Create the Set G of all granules
3. Initialize arrays Upper(0..n) and Lower(0..n) to zero values
for i = 1 to total_no_granule do
    max(i) = maximum gray value of pixels in granule(i);
    min(i) = minimum gray value of pixels in granule(i);
    foreach Interval T(i) of Int do
        if < min(i), max(i) > contained completely in T(i) then
            Lower(i)++; Upper(i)++;
        end
        else if < min(i), max(i) > contained partially in T(i) then
            Upper(i)++;
        end
    end
end
for l = 1 to n do
    roughness(l) = 1 - [ L(l) / U(l)];
end
for l = 1 to n do
    Rough_entropy = Rough_entropy -[e/2] × [roughness(l) ×
    log(roughness(l));
end
```

3.3 Proposed 2D $MRET$ algorithm

Standard 1D $MRET$ algorithm as described by Malyszko and Stepaniuk (2008) finds N-thresholds for further thresholding according to this thresholds, creating segmentations of processed images. This algorithm has been investigated in the area of one-dimensional thresholding, meaning that thresholded image consisted of one image, that was subjected to algorithm flow. However, taking into account more precessed images with different features of the same image scene should improve segmentation quality. Solutions of this kind are frequently met during segmentation of satellite imagery, when images with several bands are available. The paper presents extension of granular $1DMRET$ algorithm into segmentation of simultaneous two images referred to as $2D$ - dimensional thresholding. Two-dimensional $MRET$ algorithm makes use of two separate 1D dimensional images.

In Algorithm 1, 3, 4, 5 procedures for calculating Rough Entropy Measure for given input intervals sorted in ascending order are given together with managing iterative evolutionary population in search of optimal solutions. In this regard, population of S separate solutions in the form of two threshold series each is created, and procedure given in Algorithm 1 is performed until termination criteria are met. In each iteration, data structures initialization proceeds as described in Algorithm 3. For the given two thresholds series in each dimension, two-dimensional ranges are created. Two two-dimensional arrays for storing lower and upper approximations Lower, Upper are zeroed. Next, for each granule two-dimensional mean value and standard deviation value is computed. Further, depending whether mean value(1) + standard deviation(1) is contained in given range assignment to appropriate Lower and Upper approximation is performed. If calculated value is completely contained in given range T, both approximations are incremented,otherwise only Upper approximation is incremented. Consequently, three further calculation are performed in the same way for: mean value(1) + standard deviation(1), mean value(2) - standard deviation(2), mean value(2) + standard deviation(2). In the calculations, mean value(1) represents mean value in the first dimension, mean value(2) represents mean value in the second dimension. The same naming convention applies for standard deviation values. After all granules have been processed, rough entropy measure is calculated according to routine given in Algorithm 5. On the base of calculated rough entropy measure, each solution undergoes selection, cross-over, mutation operation and elite solutions are additionally retained in the population with the best β-index values. Next, further iterative evolutionary steps as described in Algorithm 1 are performed until termination criteria are met.

Algorithm 3: Granular Rough Entropy calculation - initialization routines

Data: Input Image, granule parameters: GX, GY, GSX, GSY ;
$T1_0, T1_1, ..., T1_{n-1}$, n Input first dimension thresholds ;
$T2_0, T2_1, ..., T2_{n-1}$, n Input second dimension thresholds ;
Result: 2D Rough Entropy Measure
1. Create two dimensional set Int of intervals generated according to Input Thresholds
$T(1,1) = < 0 - T1_0, 0 - T2_0 >, \ldots, T(n,1) = < T1_{n-1} - 255, 0 - T2_0) >$
$T(1,2) = < 0 - T1_0, T2_1 - T2_2 >, \ldots, T(n,2) = < T1_{n-1} - 255, T2_0 - T2_1 >$
$\ldots$

$T(1,n) = < 0 - T1_0, T2_{n-1} - 255 >, ..., T(n,n) = < T1_{n-1} - 255, T2_{n-1} - 255 >$
2. Create the Set G of all granules
3. Initialize arrays $Upper(0..n, 0..n)$ and $Lower(0..n, 0..n)$ for each generated interval $T(.,.)$ to zero values

4 Experiments

Experiments have been carried out for three images sets: *Lenna* images, $IKONOS$ images and $TEXT$ images. The set of *Lenna* images consisted from four images standard image, and three convoluted images with the window $3x3$ with operations min, max and mean. The set of $IKONOS$ images consisted from four images in Red, Green, Blue and InfraRed bands. The set of $TEXT$ images consisted from

Algorithm 4: Granular Rough Entropy - calculation of Lower and Upper Approximations

```
for i = 1 to total_no_granule do
    mean(i, 1..2) = mean gray value of pixels in granule(i);
    std(i, 1..2) = standard deviation of gray values of pixels in granule(i)
    separately for first and second image;
    foreach Interval T(i, j) of Int do
        foreach k=1,2 do
            if < mean(i, k), mean(i, k) − std(i, k) > contained completely in
            T(i, j) then
                Lower(i,j)++; Upper(i,j)++;
            end
            else if < mean(i, k), mean(i, k) − std(i, k) > contained partially
            in T(i) then
                Upper(i,1)++;
            end
            if < mean(i, k), mean(i, k) + std(i, k) > contained completely in
            T(i, j) then
                Lower(i,j)++; Upper(i,j)++;
            end
            else if < mean(i, k), mean(i, k) + std(i, k) > contained partially
            in T(i) then
                Upper(i,1)++;
            end
        end
    end
```

standard image, and three convoluted images with the window $3x3$ with operations min, max and $mean$. $IKONOS$ images consisted from images in the bands R, G, B and IR. $TEXT$ images consisted from standard image and its three convoluted images with the window $3x3$ with operations min, max and $mean$. Four $Lenna$ images (std, min, $max, mean$) have been independently thresholded by means of 2D $MRET$ algorithm. In each experiment as input images pairs of images $Std - Min$, $Std - Max$, $Std - Mean$, $Min - Max$, $Min - Mean$, $Max - Mean$ have been taken as input. $IKONOS$ images (R, G, B, IR) have been independently thresholded by means of 2D $MRET$ algorithm. In each experiment as input images pairs of images $R - G$, $R - B$, $R - IR$, $G - B$, $G - IR$, $B - IR$ have been taken as input. $TEXT$ images $(std, min, max, mean)$ have been independently thresholded by means of 2D $MRET$ algorithm. In each experiment as input images pairs of images $Std - Min$, $Std - Max$, $Std - Mean$, $Min - Max$, $Min - Mean$, $Max - Mean$ have been taken as input. For each experiment, population of 30 solutions has been created, and evolutionary algorithm has been iterated 30 times according to steps given in Subsection 3.3. During evolutionary routines, solutions for selection have been chosen that resulted in the best fitness values - rough entropy measure in case of 2D $MRET$ algorithm and 2D $Otsu$ measure in

Algorithm 5: 2D Rough Entropy - roughness and roughness entropy

for k = *1* **to** n **do**
    for l = *1* **to** n **do**
        roughness(k,l) = 1 - [Lower(k,l) / Upper(k,l)];

for k = *1* **to** n **do**
    for l = *1* **to** n **do**
        Rough_entropy = Rough_entropy $-[\frac{e}{2}] \times$ [roughness(k,l) $\times$ log(roughness(k,l));

case of 2D *Otsu* thresholding algorithm. Mutation rate was set to 0.20%. Cross-over probability was 100%, meaning that all selected solutions undergo cross-over operation. For 2D *MRET* algorithm results, separately *beta*-index values for each dimension are presented as 1D First and 2D Second column, next *beta*-index value for 2D image is presented, and the last column contains value of 2D rough entropy measure. For 2D *Otsu* algorithm results, 1D First and 1D Second image *beta*-index values are given, next *beta*-index of 2D image is given. Column 2D *Otsu* presents 2D *Otsu* variance value for 2D image.

Lenna **images**. In Table 1 index-β values of the solutions from 2d *MRET* algorithm runs for $R = 5$ ranges are presented. In each algorithm run, sizes and shifts of granules are equal G(2,2,2,2), it means that granules sizes are $2x2$ and granules are shifted by $2x2$ step. In Table 2 index-β values and 2D *Otsu* variance of the solutions from 2d *Otsu* thresholding algorithm runs for $R = 3$ ranges are presented. In Table 1 solutions that yielded better results than 2D Otsu algorithm based segmentations are bolded.

TABLE 1: Quality Indices for 1D Image *Lenna* for 2d*MRET*

Image Lenna	1D First β	1D Second β	2D β	2D RE
Std-Max	**24.80**	19.86	37.31	9.15
Std-Min	**25.93**	**30.17**	**42.99**	8.13
Std-Mean	22.31	28.00	51.47	4.27
Max-Min	26,55	**34.41**	**39.95**	8.95
Max-Mean	22.15	23.00	40.09	7.38
Min-Mean	**32.95**	26.05	**48.04**	8.54

IKONOS **and** *TEXT* **images**. In Table 3 and Table 5 index-β index-β values of the solutions from 2d *MRET* algorithm runs for $R = 3$ ranges are presented for *IKONOS* and *TEXT* images. In each algorithm run, sizes and shifts of granules are equal G(2,2,2,2), it means that granules sizes are $2x2$ and granules are shifted by $2x2$ step. In Table 4 and 6 index-β values and 2D *Otsu* variance of the solutions from 2d *Otsu* thresholding algorithm runs for $R = 3$ ranges are presented. In Table 3 and 5 solutions that yielded better results than 2D Otsu algorithm based segmentations are bolded.

TABLE 2: Quality Indices for 1D Image *Lenna* for 2d*MRET*

Image Lenna	1D First β	1D Second β	2D β	2D Otsu
Std-Max	22.78	27.27	42.18	97.70
Std-Min	19.68	30.07	41.56	109
Std-Mean	24.92	28.28	54.96	76.21
Max-Min	27.76	33.91	39.26	109.02
Max-Mean	27.29	28.02	44.05	88.48
Min-Mean	29.35	29.54	47.78	91.10

TABLE 3: Quality Indices for 2D *IKONOS* images for 2d*MRET*

Image IKONOS	1D First β	1D Second β	2D β	2D RE
R-G	**4.99**	**6.50**	**14.64**	3.00
R-B	7.87	16.71	16.28	6.03
R-IR	**6.57**	**9.04**	**12.92**	4.72
G-B	8.22	**17.29**	**15.12**	6.47
G-IR	6.06	**8.58**	14.67	2.99
B-IR	17.48	**8.72**	**15.37**	6.29

TABLE 4: Quality Indices for 2D *IKONOS* images for 2d *Otsu*

Image IKONOS	1D First β	1D Second β	2D β	2D Otsu
R-G	3.34	4.55	14.50	72.25
R-B	7.97	17.28	16.36	175.00
R-IR	5.82	6.79	12.01	123.80
G-B	8.18	17.28	14.98	213.48
G-IR	6.20	8.35	14.95	121.80
B-IR	17.50	8.70	15.31	237.50

5 Conclusions and Further Research

In the paper, new algorithmic approach to two-dimensional multilevel thresholding scheme has been proposed. Two-dimensional multithresholding based on maximizing rough entropy measure is presented. Proposed 2D thresholding solution presents extension of 1D multithresholding $MRET$. Experimental results have proved usefulness of the $MRET$ algorithm in the area of 2D thresholding with segmentation quality comparable and most often outperforming reference 2D *Otsu* thresholding scheme. Future research is possible into further investigation of proposed 2D rough entropy thresholding scheme in clustering domain.

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Table 5: Quality Indices for 2D $TEXT$ images for 2d $MRET$ thresholding

Image TEXT	1D First β	1D Second β	2D β	2D RE
Std-Min	**45.75**	**96.23**	82.02	3.50
Std-Max	**47.74**	14.61	49.61	4.07
Std-Mean	**47.35**	**28.13**	45.44	3.16
Min-Max	**96.16**	**15.16**	**91.15**	2.74
Min-Mean	95.15	**15.53**	89.55	2.95
Max-Mean	**15.09**	**26.88**	33.97	2.60

Table 6: Quality Indices for 2D $TEXT$ images for 2d $Otsu$ thresholding

Image TEXT	1D First β	1D Second β	2D β	2D Otsu
Std-Min	45.57	95.23	82.50	150.80
Std-Max	47.23	14.95	49.62	74.69
Std-Mean	44.15	27.32	45.85	124.29
Min-Max	95.98	14.80	90.83	98.72
Min-Mean	90.51	10.66	89.62	122.25
Max-Mean	14.69	26.62	34.12	65.11

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