

Associational Cognitive Maps for Medical Diagnosis Support

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Abstract

This paper presents the results of our investigation of the problem of representing knowledge for medical diagnosis systems. The main topic of the presented effort is the representation of cause-and-effect associations within medical data by the application of a conceptual model of cognitive maps. After presenting the theoretical background and some of the relative works, we introduce a new model of associational cognitive maps (ACM). The theoretical contribution of the proposed ACM model is its ability to represent and reason with the structures of causally dependant concepts. These structures can be identified as a kind of higher-level dynamical concepts within a cognitive map. The proposed model enables also complex bi-directional reasoning procedure that is applied for medical diagnosis support. Due to its easy graphical interpretation, the proposed ACM can be used to make the medical knowledge widely available through computer consultation systems.

Keywords: medical diagnosis, decision support, cognitive maps

1 Medical diagnosis problem

Let's focus our attention primarily on the process of making medical diagnoses. In medical science, patients have symptoms that prompt them to see a doctor. In other words, symptoms are comprised of the observations reported by patients and the observations of doctors while examining patients. Symptoms are manifestations of a disease or group of diseases. The identification of the underlying cause of symptoms is crucial and improves the chance of proper diagnosis of the disease and prescribing the correct treatment. Sometimes, the initial treatment must be based on symptoms alone in order to treat severe symptoms, such as excessive body temperature. Unfortunately, when this is done without treating the underlying cause, the symptoms are very likely to recur.

The first problem in making a medical diagnosis is the classification of medical data. If quantitative data are available as records of measurements (e.g., the measurement of the number of white cells in the patient's blood), there is usually a need to map them to meaningful categories (concepts). The terms "medical diagnosis" and "medical concept" (Chen *et al.*, 2005) are often understood intuitively. They involve the dynamics of causes and symptoms with their mutual positive or negative influences. The classification of quantitative data to the qualitative

medical concept depends frequently on diverse factors, e.g., factors related to the patient's age or the history of the disease. The analysis of the representation of medical concepts and the modeling relationships among them have been presented in chapters 6 and 7, respectively, of the noted reference (Chen *et al.*, 2005).

As we have sketched above, the problem of medical diagnosis requires the construction of a complex representation scheme to capture static and dynamical phenomena that can be encountered during the examinations of a patient. We would like to emphasize that the key feature of such knowledge representation is its capability of precise representation of diverse types of data and associations within them. This involves:

1. qualitative or quantitative description of symptoms and their classification to the related medical concepts,
2. representation of causal relationships between medical concepts, and
3. the possibility of backward reasoning, i.e., identification of causes on the basis of symptoms.

2 Medical decision support systems – related works

During the last thirty years, an enormous number of decision support systems (DSS) for diverse medical problems has been developed. Some of these systems are easy to find on the Internet, and they are freely available for public use. The traditional medical expert systems (Hudson, 2006), were equipped with a rule knowledge base supplied by the domain experts (doctors). On the basis of rules inserted in the expert system, it is possible to classify new instances of medical observations by matching symptoms to the conditional part of a rule and then to perform forward reasoning to achieve the diagnosis or construct a therapy plan. In our opinion, one of the main disadvantages for the application of the classic rule-based knowledge representation in medical DSS is its limitation of representing some of the more complex associations that may be experienced within the medical data. For example, in a rule-based DSS, the representation of the complex phenomenon of causality (Pearl, 2000) is, in fact, left to the interpretation and expertise of the doctor. Formally, it is reduced to drawing purely logical inference about a decision attribute on the basis of the co-occurring values of conditional attributes. In fact, the situation can be much complex. Let consider the logical implication “if A, then B.” On the basis of the “modus ponens” rule, if A is true, we can conclude that B is also true, but we cannot draw this inference in the case in which A is false. However, assuming proper semantics, from the statement “A causes B,” the inference “if A does not occur, then B is absent” is possible. The letter A may stand for the disease and B for the symptom. The attempts to overcome some of the limitations of the rule-based system has led, for example, to the annotation of rules in natural language or the incorporation of non-classical logic to the DSS.

On the opposite site of the attribute-based approach lies the conceptual modeling approach. The main idea of conceptual knowledge representation comes from the biology of the human mind and cognitive science. It is inspired by biological and psychological observations that human knowledge is structured in the form of

concepts and associations between them. The biological inspirations led initially to the general and intuitive understanding of such terms as ‘concept’ or ‘associations among concepts,’ which later were assigned different meanings in the computer science literature. There is a vast number of knowledge-representation methods that can be considered, in general, as exemplification of the conceptual modeling approach. Maybe the best-known approach to conceptual modeling is represented by ontologies and semantic networks that are able to express concepts and relationships among them. In medical science, we can mention the ontology of UMLS (Unified Medical Language System) (McCray, 2003). It provides a unified vocabulary of terms used in medicine and the set of relationships as building blocks for the developed medical ontology.

Maybe less known in computer science are cognitive maps (CMs). Cognitive maps were introduced to the social sciences by Axelrod (1976). In the artificial intelligence domain, CMs can be classified to the connectionist knowledge representation methods, the same group as neural networks, Bayesian belief networks, semantic networks, ontologies and other. Despite some inspirations borrowed from their competitors, CMs differ in many features.

3 Cognitive maps as a knowledge representation scheme

The initial theoretical model (Axelrod, 1976) of CMs was targeted at the representation of cause-and-effect relationships among concepts described in natural language. The intuitive and easy-to-interpret representation of a CM is a directed graph, consisting of concepts nodes and cause-and-effect relationships (directed arcs) among them. The concepts represent states that are observable within the domain of interest. In a case a particular concept has been observed, it is assumed as activated and can afterwards influence the activation of other concepts in the graph. This means that concepts can be activated by means of observation or by the stimulation achieved from their neighbors in the graph of the CM. The arcs indicate the directions of casual dependencies between source and target concepts. In the generic model of a CM, the causal dependencies can be positive (the respective arc is labeled with a “+” sign), i.e., the activation of one concept causes the activation of the other, or negative (the respective arc is labeled with “-” sign), i.e., the activation of a concept can deactivate the other one. The dynamics of CMs is thus expressed by arcs that can be used to represent and to simulate the behavior of the CMs over time. Note that the initial intuitive model of the CM used in social sciences and economics did not define precisely many important features of the CM, e.g., what does it precisely mean that a concept is active, or what does a causal relationship among concepts mean formally.

Among many extensions to the generic CM model, the most influential are the fuzzy cognitive maps (FCMs), introduced by Kosko (1986). The concepts in this model are mapped to the continuous interval $a_i \in [0, 1] \in \mathbb{R}$, where 0 means no activation and 1 means full activation. The arcs in this model are labeled by weights $w_{i,j} \in [-1, 1] \in \mathbb{R}$, where -1 means fully negative and +1 fully positive causal influence. Having an activation state of concepts (with only a subset of them active at the same time), it is possible, on the basis of arcs, to compute

iteratively the activation values in the future. Among many extensions of FCM we would like to mention the proposal of HO-FCMs (Stach *et al.*, 2006) that overcomes the problem of modeling high-order dynamics by adding memory to the concepts nodes. A survey of FCM extensions is presented in (Aguilar, 2005).

FCMs have already been applied in medical diagnosis support. The solution presented in (Georgopoulos *et al.*, 2003) assumes every medical diagnosis as a single concept in CM. The reasoning is performed forward, towards the stated goal of achieving the activation of one of the expected diagnosis nodes. The decomposition of FCM to the group of local maps has been proposed in (Stylios and Georgopoulos, 2008). The combination of FCM and case-base reasoning (CBR) in medical decision support systems seems to be very promising approach (Georgopoulos and Stylios, 2005). The detailed comparison our ACM model to the existing solutions is beyond the scope of this paper.

4 Associational cognitive maps

For the targeted medical diagnosis support system, we propose a new ACM model of cognitive map. The proposed extension to a generic model of CM enables to represent a phenomenon that is known in medical science, i.e., the existence of symptoms that are indirectly associated within a particular causal structure (that can be unknown to a doctor during the diagnosis). We start defining the ACM model from the notion of concept.

4.1 Concepts

The concept and its later practical construction are always related to the problem environment. Let U be a finite set of observations $u \in U$. A concept will be understood as the subset of observations $c \subseteq U$. The set of all concepts will be denoted as C . Intuitively, every concept is a class to which the observations from U are mapped. Let us define a parameterized observation(classification) function:

$$\psi_{\bar{p}} : U \times C \longrightarrow [0, 1], \quad (1)$$

where \bar{p} is a vector of parameters, and $[0, 1] \in \mathbb{R}$. It is possible now to classify the observation value $u \in U$ to the particular class $c \in C$ in an approximated way (e.g., using fuzzy or rough (Skowron and Stepaniuk, 2001) membership functions). In particular, note that assuming c as a fuzzy set and interpreting $\psi_{\bar{p}}(u, c)$ as a fuzzy membership function do not automatically imply the future application of classical fuzzy set operators or fuzzy logic while performing reasoning within concept space C . The observation function can be decomposed with respect to concepts and then considered as a vector $\bar{\psi}$. The separate vector of parameters \bar{p} for every function within $\bar{\psi}$ is usually useful in many classification methods and can involve, for example, the quality of the approximation threshold or the length of the so-called time window used in the construction of temporal concepts. It enables also the generation of diverse (optimized) versions of concepts by modifying the values of parameters (Froelich and Wakulicz-Deja, 2007).

Similarly, as in the generic FCM model, let us now define a mapping $a : C \rightarrow [0, 1]$. Thus, in particular, on the basis of $\psi_{\bar{p}}(u, c)$ it is possible to compute the

so called activation value $a(c_i) \in [0, 1]$ for every concept $c_i \in C$. For simplicity, $a(c_i)$ will be denoted as a_i . Considering the activation values of all concepts of the ACM, a corresponding vector \bar{a} can be constructed. If the time has to be considered, the respective objects (concepts, observation functions, and activation values) will be complemented by the letter t .

4.2 Causal relationships

In our ACM, we would like to propose a generalized representation scheme of causal knowledge within a cognitive map. Let $\kappa \subseteq C \times C$ be a binary relation 'is-a-cause' within C :

$$c_1 \kappa c_2 \text{ -- denotes: } c_1 \text{ is a cause of } c_2 \quad (2)$$

Since the relation κ should reflect human causal knowledge (e.g. in a medical diagnosis problem), it should possess some of features experienced in common sense reasoning. It should also impose some constraints on the construction of our ACM. For the two pair-wise, exclusive (in the sense of corresponding subsets of observations) concepts c_1, c_2 and a given time period Δt we assume the following axioms:

1. $\forall c_1 \in C. \neg(c_1 \kappa c_1)$ – anti-reflexivity
2. $\forall c_1, c_2 \in C. c_1(t) \kappa c_2(t + \Delta t) \Rightarrow \neg(c_2(t) \kappa c_1(t + \Delta t))$ – anti-symmetry

Axiom 1 assumes that a concept cannot be the cause of itself. Due to the incorporation of the time variable, axiom 2 needs some explanation. If a particular concept c_1 is a cause of c_2 , then the reverse situation is not possible at the same time, but can occur in a later period of time. One can talk about causal relations expressed in a formal way, but the interpretation and identification of the relations in real life are much harder (Pearl, 2000). Let us now consider possible interpretations of our κ relation. We would like to propose its interpretation in the problem environment in general way, by applying the characteristic function μ . We write: $c_1 \kappa c_2 \Leftrightarrow \mu(c_1, c_2) \in Z$.

The set Z is not restricted to numerical values, e.g., $Z = [-1, 1]$ and can involve linguistic utterances, e.g., “weak” or “strong”. Note, that within our assumptions, the method of computation of μ is not necessarily the same for all concepts and is not restricted to taking into account only the changes of activation values of two concepts in consecutive moments of time. The analysis of possible learning methods of ACM is a challenge beyond the scope of this paper. Depending on the available data (type of environment, e.g., observational, temporal or experimental), the attempts for proper construction of μ and thus interpretation of κ can be performed in diverse ways:

- by expert, e.g., by means of a-priori ontology containing κ -relation,
- by the application of statistical methods with constraints (Pearl, 2000),
- by the analysis of temporal data, and
- by experimenting in dynamic environment.

Since medical knowledge comes from long years of study and experience, in our opinion, the main part of the map should be constructed or at least verified by experts.

4.3 Associational structures within ACM

Let 2^C be the family of all subsets of C (the power set of C), and let $D \subset 2^C$ be its non-empty subset. Let T be an ordered set of time labels $\{t_0, t_1, \dots, t_n\}$. We will call $C_a \in D$ the causal association set within the space of concepts C , if both of the following conditions hold:

1. $\forall c_i \in C_a. \exists c_j \in C_a. (c_i \kappa c_j \vee c_j \kappa c_i)$
2. $\forall c_i, c_j \in C_a, c_i \kappa c_j. \exists t_s, t_e \in T, t_s < t_e. \forall t_l \in [t_s, t_e]. (a_i(t_l) > 0 \Rightarrow a_j(t_{l+1}) > 0)$

Intuitively, the first condition means that there are no causally isolated concepts within C_a . The second condition specifies that the activation of concepts within C_a may occur (at least one time t_s within considered period T) in order specified only by the isolated causal structure within C_a . Note that due to the influence of external (to the C_a) concepts, the other order of concepts' activations within the pointed substructure may occur but in this case it does not satisfy condition 2 and is not considered as the causal association set within C . The associational set specifies in fact a kind of causal substructure within ACM.

The associational cognitive map is defined as the n-tuple:

$$ACM = \langle C, \bar{\psi}, \kappa, D \rangle, \quad (3)$$

where C is the finite set of concepts, ψ is the classification function, κ is the causal relation and D is a predefined set of diagnoses (family of subsets of concepts).

4.4 Reasoning within ACM

Let us assume, that at time t_0 the activation values of all concepts are computed on the basis of the observation function, i.e. $\forall c_i \in C. (a_i(t_0) = \psi_{\bar{p}}(u, c_i))$ or $\bar{a}(t_0) = \bar{\psi}(t_0)$. Let $C_0 = \{c_i \in C : a_i(t_0) > 0\}$ be the set of active concepts. We can search for the possible effects and causes of concepts from C_0 . This can be achieved by forward and backward reasoning within ACM. For every $c_j \in C_0$ we search forward for the specific subset of effects $C_f \subset C$ such that for every $c_k \in C_f$ holds: $\forall c_k \in C_f. \exists c_j \in C. c_j \kappa c_k$. Let us activate all concepts from C_f using the formula: $\forall c_k \in C_f. (a_k = \min \left[\max \left[\sum_j (a_k + a_j \mu(c_j, c_k)), 0 \right], 1 \right])$. The computation of *min* and *max* functions serves to reduce unbounded values to a strict $[0, 1]$ range. Let us denote as $\bar{\mu}(t_i)$ the square matrix of all current values of characteristic function $\mu(c_i, c_j)$. The computation of activation values can be achieved by the matrix multiplication $\bar{a}(t_{i+1}) = \min \max \bar{a}(t_i) \bar{\mu}(t_i)$. The causal association set can be constructed this way: $C_a = C_f(t_0) + C_0$ and recursively: $C_a(t_{i+1}) = C_f(t_i) + C_a(t_i)$. There are obviously some factors that can determine stopping conditions for the reasoning process e.g.: limited scope of ACM (no more effects or causes) or going into the cycle behavior.

Similar way, we can perform backward reasoning and search for the set $C_b \subset C$ of possible causes of C_0 . For every $c_i \in C_b$ holds: $\forall c_i \in C_b. \exists c_j \in C. c_i \kappa c_j$. The

set C_b specifies all possible causes of C_0 . We activate all concepts from C_b using the formula: $\forall c_i \in C_b. (a_i = \min \left[\max \left[\sum_j (a_i + a_j \mu(c_i, c_j)), 0 \right], 1 \right])$ or by the equivalent matrix multiplication: $\overline{a}(t_{i+1}) = \min \max \overline{a}(t_i) \overline{\mu}(t_i)^T$, where $\overline{\mu}(t_i)^T$ denotes the corresponding transposed matrix. The causal association set can be constructed in the following way: $C_a = C_b(t_0) + C_0$ or applying the recursive formula: $C_a(t_{i+1}) = C_b(t_i) + C_a(t_i)$.

For the purposes of the medical diagnosis problem we assume that the set C consists of two subsets of concepts: symptoms and interventions (e.g., medicines). As it will be shown in our case study, it appears crucial for the reliable diagnose, to find the set of common causes of all currently observable symptoms of the disease. On the other hand, there are known in medicine science diseases expressed by a mixture of two or few causally independent symptoms. The analysis of these cases leads to the sophisticated evaluation of diverse possible causes and to the assumption of preferences within the space of concepts. Should we consider (while making diagnosis) causes common for all symptoms that are weakly activated or the other with strong activation level? However, our preliminary experiences shows that it is much easier for a doctor to find even few mutually independent causes but strongly associated with the observed symptoms, than a single common cause of all symptoms. How to find the common causes in the complex network of medical concepts? Let us consider this problem in our ACM. Initially we perform separate backward reasoning for every single symptom $c \in C_s$, where C_s denotes the set of initially observed symptoms i.e. we make the substitution $C_0 = C_s$. We assume the boundaries of ACM or cyclic behavior as stopping condition. After finishing the reasoning process, we find the set C_e of concepts, common for all C_a^i , i.e. such the following formula holds: $\forall c_k \in C_e. \forall i \in [1, 2, \dots, \text{card}(C_0)]. c_k \in C_a^i$. Afterward, it is necessary to switch to forward reasoning, primarily to show the doctor all the relevant causal paths from found causes to the observed symptoms, secondarily we need to exclude from the sum $\sum_i C_a^i$ all irrelevant concepts. We make a cut of the reasoning space i.e. we substitute $C = \sum_i C_a^i$ to remove concepts that has not been involved until this time in reasoning process. Then, we can start forward reasoning starting from the set C_e within the new C i.e. we make a substitution: $C_0 = C_e$. This time the reasoning process can be performed jointly for all concepts within C_e (due to the prior knowledge of common effects). As mentioned before, both reasoning procedures can be made as numerical computations (multiplication of matrixes), that speeds up the computational process in comparison to rule based DSS. Moreover, reasoning can be easily parallelized and involve large spaces of concepts.

Finally, we need to identify the achieved C_a as one of the subsets stored in D (which is in fact the repository of possible diagnoses). Of course, the classification can be also made in approximated way. The corresponding labels of the matched sets from D and the structure of concepts from the final C_a (including concepts and arcs) should be shown to the doctor.

5 Case study

The medical knowledge base is constructed in the form of an ACM cognitive map that was proposed in section 4. We have tested our system using data taken from real medical case. At the time of writing this paper, $\bar{\psi}(t)$ is very simplified and constructed on the basis of simple matching of string patterns between input text given by a doctor and corresponding labels of concepts stored in the ACM. Unfortunately, the presentation of the set of considered concepts and the entire ACM fall out beyond the scope of this paper. Therefore we decided to present only a part of it in the Fig. 1. It involves also the drugs (of the given names) that are available in pharmacies in Poland. The family of concept subsets D (diagnoses) has been constructed on the basis of available medical interventions and on the basis of the knowledge of their possible impact on state of patient. The associations between diagnoses and drugs are as follows: $d_1 = \{\text{corneal ulcer, injury}\}$ – surgical intervention, $d_2 = \{\text{corneal ulcer, allergy}\}$ – alergocon, $d_3 = \{\text{corneal ulcer, bacteria in eye}\}$ – dicortineff, $d_4 = \{\text{corneal ulcer, bacteria in eye, bacteria in throat, bacteria in blood}\}$ – biotraxon or xorimax. In the case we have chosen, a patient came to the eye specialist and reported that after few days of playing tennis he had the following symptoms: a corneal ulcer and a sore throat. The patient had reported allergic reactions to various allergens in the past. He did not have a fever. The doctor suspected three probable causes: an injury after the tennis match, an allergy based on previous inclinations of the patient, and a bacterial infection of the eye. The first proposed treatment was related to the suspected injury, i.e., a simple surgical removal of the ulcer. Because the patient denied having sustained an injury to his eye, he was given alergocon, an anti-allergy drug in the form of eye drops and dicortineff, a mixture of antibiotics also in the form of eye drops. Unfortunately, all the diagnoses and treatments were wrong. After three weeks, there was no improvement in the patients symptoms. Where was the mistake? The doctor neglected to take into account the importance of the reported symptom of a sore throat. He didn't even examine the patient's throat. Subsequently, the patient went to another doctor, who conducted a bacterial examination of the patient's throat with negative results. The examination was repeated a few times without identifying any bacteria in the throat, in spite of the fact that visual examination clearly suggested a bacterial infection in the patient's throat. Even blood analyses detected no bacteria that could be causing the sore throat. In spite of the disappointing investigations, this time the diagnosis was right. There was a body-wide bacterial infection that had caused both the sore throat and the corneal ulcer. The lack of fever also suggested a bacterial infection as opposed to a viral infection, which usually causes high fever. The first proposed medicine was biotraxon, an antibiotic from the third generation of the cephalosporin group of antibiotics, but, unfortunately, this was another mistake. The patient had a severe allergic reaction to the antibiotic, including high fever, muscle pain, and rigor). Note, the patient had reported his past allergies, but this information was neglected by the doctor. The third doctor changed the prescribed medicine to xorimax, an antibiotic in the second generation group of cephalosporin, that could easily be tolerated by most patients. This turned out to be a very good choice taking into account the lack of identification of particular bacteria.

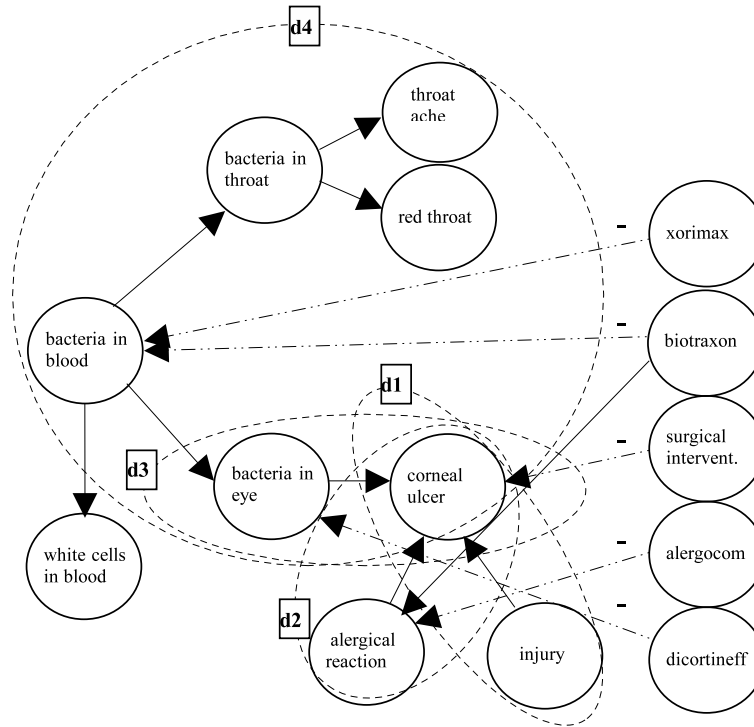


FIGURE 1: Graphical interpretation of exemplary ACM for medical diagnosis support

The previously described reasoning process were simulated in our system. After activating C_0 by the observation of initially reported symptoms (throat ache, corneal ulcer), backward reasoning was performed until the common causes of all symptoms were found. Assuming the proper initialization of observed symptoms, the ACM based system found the right diagnosis (d_4) taking into account the common cause: bacteria in blood. Note that the wrong diagnoses (d_1, d_2, d_3) are also marked in Fig. 1.

6 Conclusions

In the presented research, we have proposed the associational extension and a formal model of cognitive maps called ACM. We introduced in ACM the possibility to associate causally dependant concepts in functional groups (association sets) that can be identified as a kind of higher-level, dynamical concepts within a cognitive map. The set of associated concepts specified in general as local patterns of activation flow, should be pointed by an expert or learned automatically from data. The latter problem is the challenge for future research. The presented solution has been raised by some of the requirements imposed by the targeted application: the causal association of disease symptoms that seems to be crucial for the right medical diagnosis. We have also sketched in this paper the exemplary problem of

medical diagnosis and its simulation using the proposed solution.

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